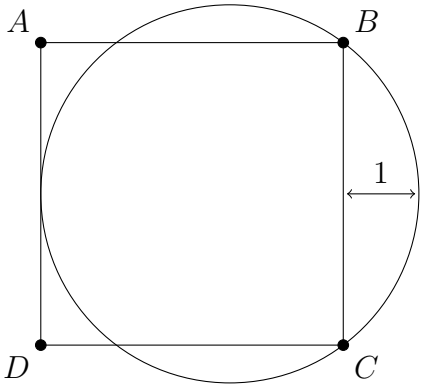
New Zealand Mathematical Olympiad Committee

Maths Workshop

November 2020

Problems

1. A circle passes through vertices B and C of square $ABCD$. The circle is also tangent to line AD. The diameter of the circle is 1 longer than the sidelength of the square. What is the area of the square?



2. You are given three squares of dimensions 2×2 and 3×3 and 6×6. Choose two of them to cut into two pieces each, such that the obtained five shapes can be arranged to assemble a 7×7 square.

3. Show that for any positive integer n, we have

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{n^2+n} < 1.$$

4. What is the smallest positive integer n, which is one more than triple the number obtained by reversing the digits of n?

5. Let p, q, x, y be any four positive real numbers. Prove that

$$\left(p^3+q^3\right)\left(x^3+y^3\right)^2 \geq \left(px^2+qy^2\right)^3$$

6. A box has a height of 64 units and a width of 27 units. The box is flush against the wall and the ground. Find the length of the shortest straight ladder which can be placed on the ground leaning against the wall, such that the ladder just touchs the corner of the box.