New Zealand Mathematical Olympiad Committee

Sample Geometry Problems
by Ross Atkins

1. A pair of circles intersect at points A and B. A line is tangent to both circles, at points C and D. Prove that the intersection of AB and CD is the midpoint of CD.

2. Let $ABCD$ be a square and let P be a point inside $ABCD$ such that $AP = BP$ and $\angle APB = 150°$. What is $\angle CPD$?

3. Let ABC be a triangle with $\angle CAB > 45$ and $\angle CBA > 45$. Construct an isosceles right angled triangle RAB with AB as its hypotenuse and R inside ABC. Also construct isosceles right angled triangles ACQ and BCP having AC and BC respectively as their hypotenuses and lying entirely outside ABC. Show that $CQRP$ is a parallelogram.

4. Consider an equilateral triangle ABC. Let P be an arbitrary point on the shorter arc AC of the circumcircle of ABC. Show that $PB = PA + PC$.