New Zealand Mathematical Olympiad Committee

NZMO Round Two 2026 — Instructions

Exam date: 11th *September*

General instructions:

1. You have 3 hours to work on the exam.
2. There are 5 problems, each worth equal marks. You should attempt all 5 problems. You may work on them in any order.
3. Geometrical instruments (ruler and compasses) may be used. Calculators, phones, computers and electronic devices of any sort are not permitted.
4. Write your solutions on the paper provided. Let the supervisor know if you need more. Write your name at the top of every page as you start using it.
5. Full written solutions — not just answers — are required, with complete proofs of any assertions you make. Your marks will depend on the clarity of your mathematical presentation. Work in rough first, and then draft a neat final version of your best attempt.
6. Make sure you fill in your details on the Declaration Form. Hand in all of your rough work, in addition to your neat solutions. The declaration form is used as a cover sheet for your submission.
7. At the end of the exam, remain seated quietly until all scripts have been collected and the supervisor indicates that you are free to move.
8. You may not take the question paper from the exam room.
9. The contest problems are to be kept confidential until they are posted on the NZMOC webpage www.mathsolympiad.org.nz. Do not disclose or discuss the problems online until this has occurred. Typically this will be approximately one week after the exam has been sat.
10. **Do not turn over until told to do so.**

NZMO Round Two 2026 — Problems

Exam date: 11^{th} *September*

- There are 5 problems. You should attempt to find solutions for as many as you can.
- Solutions (that is, answers with justifications) and not just answers are required for all problems, even if they are phrased in a way that only asks for an answer.
- Read and follow the "General instructions" accompanying these problems.

Problems

1. Find all positive integers m and n such that
$$\frac{5m+n}{3m+7n}$$
is a fraction in its simplest form (i.e. it cannot be reduced further).

2. Let ABC be an acute triangle with $AB < AC$. Let D and E be the respective feet of the altitudes from A to BC and B to AC, and suppose that $DC = 3 \cdot BD$. Let X be the reflection of point E over line AD. Prove that $AXBC$ is cyclic.

3. Do there exist real numbers x, y and z satisfying the following?
$$x^4 + (y-2)^2 = y^4 + (z-2)^2 = z^4 + (x-2)^2 = \frac{7}{4}.$$

4. Let $\mathbb{Q}^+$ denote the set of positive rational numbers. Does there exist an infinite subset $\mathcal{S}$ of $\mathbb{Q}^+$ such that every positive rational number is the product of the elements of a unique finite subset of $\mathcal{S}$?

   Note: We consider the product of the empty set $\{\}$ to be 1.

   For example, if $\mathcal{S} = \{\frac{1}{2}, 3, 5, 7, 9, 11, 13, 15, \dots\}$, then 9 would be a product of a unique finite subset of $\mathcal{S}$, but not $\frac{15}{2}$, as it can be made by either $\frac{1}{2} \times 15$ or $\frac{1}{2} \times 3 \times 5$.

5. Let p_i denote the i^{th} odd prime, so $p_1 = 3$, $p_2 = 5$, etc. Does there exist an infinite sequence of positive integers $a_1, a_2, a_3, \dots$ such that for all positive integers n, the number $a_1^2 + a_2^2 + \dots + a_n^2$ is a multiple of p_n?