New Zealand Mathematical Olympiad Committee

NZMO Round One 2026 — Instructions

Submissions due date: 22^{nd} July

The **New Zealand Mathematical Olympiad** (NZMO) consists of two rounds:

Round One (the NZMO1): A take home exam (the following set of 9 problems). Solutions are to be submitted by 22^{nd} *July* (21:00 NZST).

Round Two (the NZMO2): A three hour supervised exam in August.

Participants in the NZMO1 will receive an indication of whether they have been invited to participate in the NZMO2; participants in the NZMO2 will be told the level of their award (gold/silver/bronze/honourable mention), if any. Scores will not be announced for either round.

In addition, the results of both rounds of the NZMO will be used to select students to participate in the training camp in January 2027 and the female and non-binary camp in May 2027. For more information on awards and eligibility please see the regulations:

https://www.mathsolympiad.org.nz/competitions/nzmo/nzmo-regulations.pdf

General instructions:

- Participation in the NZMO is open to any student enrolled in the New Zealand education system, at secondary school level or below.
- The NZMO is an individual competition. Participants must work on the problems entirely on their own, without assistance from anyone else.
- Electronic devices may not be used to assist in solving the problems. This includes but is not limited to calculators, computers, tablets, smart phones and smart watches.
- As an exception to the above, the internet may be used as a reference for looking up definitions.
- Although some problems may seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided.
- Solutions must be handwritten and then scanned or photographed. Typed solutions are strictly forbidden.
- If you have trouble scanning your work please email info@mathsolympiad.org.nz
- You must keep a record of ALL rough work used to make progress towards a problem. These must be scanned and submitted alongside your solutions. These will never negatively affect the score awarded.
- It is forbidden to discuss the problems until the official solutions are posted on the NZMOC website. Typically this will be 3 days after the submission deadline.
- Please address **all** queries to Kevin Shen, info@mathsolympiad.org.nz

The NZMO1 submission url is: https://www.mathsolympiad.org.nz/competitions/nzmo/enter.php.

All your work should be submitted as a **single document** in PDF format. Please complete the submission form **carefully**, especially your contact details.

New Zealand Mathematical Olympiad Committee

NZMO Round One 2026 — Problems

Submissions due date: 22^{nd} July

- There are 9 problems.
- Read and follow the "General instructions" accompanying these problems.
- If any clarification is required, contact Kevin Shen (info@mathsolympiad.org.nz).

Problems

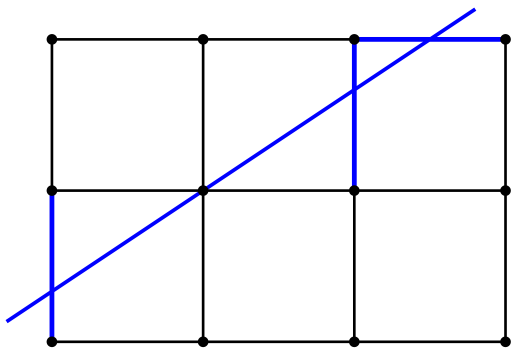
1. Determine all pairs of positive integers (m, n) such that $2^m + 3^m + 5^m = n!$.
2. Points A, B, C and D lie on a circle in that order, and I is the incentre of triangle ABD. Suppose CI bisects $\angle BCD$. Prove that triangle BCI and triangle ICD are similar.

   (The *incentre* of a triangle is the point at which its internal angle bisectors meet.)
3. Suppose real numbers a, b, c satisfy $\left(1 + \frac{a}{b}\right)\left(1 + \frac{b}{c}\right)\left(1 + \frac{c}{a}\right) = 2$. Prove that at least one of
$$\left|\frac{a+b-2c}{a+b+c}\right|, \left|\frac{b+c-2a}{a+b+c}\right|, \left|\frac{c+a-2b}{a+b+c}\right|$$
   is smaller than or equal to $\sqrt[3]{2}$.
4. A rectangle R is divided into a set $\mathcal{S}$ of finitely many smaller rectangles with sides parallel to the sides of R, such that no four rectangles in $\mathcal{S}$ share a common corner. An ant starts at a corner of a rectangle in $\mathcal{S}$, and on each move, is allowed to choose a rectangle r in $\mathcal{S}$ such that the ant is currently at one of the corners of r and jump to the opposite corner of r. Show that, regardless of how R is divided into smaller rectangles, there will always be two corners in $\mathcal{S}$ that the ant will not be able to jump between.
5. Do there exist real numbers x, y and z satisfying the following system of equations?
$$\begin{cases} \dfrac{1}{x+y} + \dfrac{1}{y+z} + \dfrac{1}{z+x} = 4 \\ \dfrac{z}{x+y} + \dfrac{x}{y+z} + \dfrac{y}{z+x} = 1 \\ \dfrac{xy}{x+y} + \dfrac{yz}{y+z} + \dfrac{zx}{z+x} = -1 \end{cases}$$

6. Let ABC be a triangle, and let M be the midpoint of BC. Let X be a point on segment AM. Let Y be the foot of the altitude from X to BC. Let Z be a point on segment XY. Let D and E be the feet of the altitudes from Z to AB and AC, respectively. Prove that if X lies on segment DE, then AZ bisects $\angle BAC$.

7. An $a \times b$ square grid has ab vertices (points at the corners of the squares) and $2ab - a - b$ edges joining the vertices. We draw n straight lines crossing some edges of the grid. The lines could be parallel to the sides of the grid, or at an angle. If a line only touches an edge at a vertex, it does not count as "crossing" that edge. Find, in terms of a and b, the minimum value of n such that it is possible to draw the lines in such a way that each edge in the grid is crossed at least once.

   For example, the below is an example of a line through a 4×3 grid, along with the edges it "crosses".



8. Call a positive integer N *happy* if N divides $2^{N-1} - 1$. Do there exist infinitely many pairs of primes (p, q) such that $N = pq$ is *happy*?

9. Consider the region $\mathcal{R}$ of points (x, y) in the plane such that $|xy| \leqslant 1$. Let Q be a non-self-intersecting quadrilateral such that every point on the boundary of (and inside) Q is in $\mathcal{R}$. Is it possible that Q has area greater than 8?