New Zealand Mathematical Olympiad Committee

NZMO Round One 2024 — Instructions

Submissions due date: 1st July

The **New Zealand Mathematical Olympiad** (NZMO) consists of two rounds:

Round One (the NZMO1): A take home exam (the following set of 8 problems). Solutions are to be submitted by 1st *July* (21:00 NZST).

Round Two (the NZMO2): A three hour supervised exam in August.

Awards for the NZMO (gold/silver/bronze/honourable mention) will be made based on the results of the NZMO2. Scores will not be announced for either round. Participants in the NZMO1 will only receive an indication of whether they have been invited to participate in the NZMO2; participants in the NZMO2 will just be told the level of their award, if any.

In addition, the results of both rounds of the NZMO will be used to select about 25 students to participate in the training camp in January 2025, and around 20 female or non-binary students to attend the training camp in April 2025. Only students who are New Zealand citizens or permanent residents, and who will still be enrolled in school in 2025, are eligible for the January training camp. Only students who attend this camp are eligible for selection to represent New Zealand at the 2025 International Mathematical Olympiad, (IMO).

General instructions:

- Participation in the NZMO is open to any student enrolled in the New Zealand education system, at secondary school level or below.
- The NZMO is an individual competition. Participants must work on the problems entirely on their own, without assistance from anyone else or any kind of calculator/computer (other than purely for word processing purposes). This includes but is not limited to using calculations, graphing functions, or using tools such as geogebra.
- Electronic devices may not be used to assist in solving the problems. This includes but is not limited to calculators, computers, tablets, smart phones and smart watches.
- The internet may not be used, except as a reference for looking up definitions.
- Although some problems may seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided.
- Submitting rough working and/or partial solutions cannot negatively affect the score awarded. We encourage you to submit any rough work or partial solutions.
- It is forbidden to discuss the problems until the official solutions are posted on the NZMOC website. Typically this will be 3 days after the submission deadline.
- Please address **all** queries to Dr Ross Atkins, info@mathsolympiad.org.nz

The NZMO1 submission url is: https://www.mathsolympiad.org.nz/nzmo1_submission. If you are having trouble loading the submission form, please try logging out of your google account, or opening the form in an incognito window. All your solutions and partial solutions should be submitted as a **single document** in PDF format. Please complete the submission form **carefully**, especially your contact details.

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NZMO Round One 2024 — Problems

Submissions due date: 1st July

- There are 8 problems.
- Read and follow the "General instructions" accompanying these problems.
- If any clarification is required, contact Dr Ross Atkins (info@mathsolympiad.org.nz).

Problems

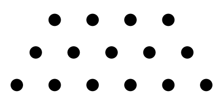
1. Josie and Kevin are each thinking of a two digit positive integer. Josie's number is twice as big as Kevin's. One digit of Kevin's number is equal to the sum of digits of Josie's number. The other digit of Kevin's number is equal to the difference between the digits of Josie's number. What is the sum of Kevin and Josie's numbers?

2. Prove the following inequality

$$\frac{6}{2024^3} < \left(1-\frac{3}{4}\right)\left(1-\frac{3}{5}\right)\left(1-\frac{3}{6}\right)\left(1-\frac{3}{7}\right)\dots\left(1-\frac{3}{2025}\right).$$

3. A rectangular sheet of paper is folded so that one corner lies on top of the corner diagonally opposite. The resulting shape is a pentagon whose area is 20% one-sheet-thick, and 80% two-sheets-thick. Determine the ratio of the two sides of the original sheet of paper.

4. A dot-trapezium consists of several rows of dots such that each row contains one more dot than the row immediately above (apart from the top row). For example here is a dot-trapezium consisting of 15 dots, having 3 rows and 4 dot in the top row.



A positive integer n is called a trapezium-number if there exists a dot-trapezium consisting of exactly n dots, with at least two rows and at least two dots in the top row. How many trapezium-numbers are there less than 100?

5. A shop sells golf balls, golf clubs and golf hats. Golf balls can be purchased at a rate of 25 cents for two balls. Golf hats cost \$1 each. Golf clubs cost \$10 each. At this shop, Ross purchased 100 items for a total cost of exactly \$100 (Ross purchased at least one of each type of item). How many golf hats did Ross purchase?

6. Let ω be the incircle of scalene triangle ABC. Let ω be tangent to AB and AC at points X and Y. Construct points X' and Y' on line segments AB and AC respectively such that $AX' = XB$ and $AY' = YC$. Let line CX' intersects ω at points P, Q such that P is closer to C than Q. Also let R be the intersection of lines CX' and BY'. Prove that $CP = RX'$.

7. Some of the 80960 lattice points in a 40×2024 lattice are coloured red. It is known that no four red lattice points are vertices of a rectangle with sides parallel to the axes of the lattice. What is the maximum possible number of red points in the lattice?

8. Let a, b and c be any positive real numbers. Prove that

$$\frac{a^2+b^2}{2c} + \frac{a^2+c^2}{2b} + \frac{b^2+c^2}{2a} \geqslant a+b+c.$$