New Zealand Mathematical Olympiad Committee

2011 June Problems

These problems are intended to help students prepare for the 2011 camp selection problems, used to choose students to attend our week-long residential training camp in January.

The solutions will be posted in about one month's time, but can be obtained before then by email if you write to one of us with evidence that you've tried the problems seriously.

Good luck!

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1. Find the least integer k for which the number 2010 can be expressed as the sum of k square integers.

2. Find all real numbers a, b, c which satisfy

$$a + 2b + 3c = 12,$$
$$2ab + 3ac + 6bc = 48.$$

3. A bucket contains n black balls and m white balls. Every turn you take two balls out: if they are the same colour, remove them and add a black ball to the bucket. If they have different colours, put the white ball back and remove the black one. What is the probability that the last ball is black?

4. If a parallelogram $ABCD$ is such that the angle bisector of $\angle BAD$ intersects the segment BC at its midpoint M, find $\angle AMD$.

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