New Zealand Mathematical Olympiad Committee

Camp Selection Problems 2011 — Instructions

Solutions due date: 10 August 2011

These problems will be used by the NZMOC to select students for its International Mathematical Olympiad Training Camp. The camp will be held in Auckland between the 8th and 14th of January 2012. Only students who attend this camp are eligible for selection to represent New Zealand at the 2012 International Mathematical Olympiad (IMO), to be held in Argentina in July 2012. The cost of the camp is yet to be determined precisely, but will not be more than $500.

At the camp a squad of 10–12 students will be chosen for further training, and to take part in several international competitions, including the Australian and Asia-Pacific Mathematical Olympiads. The New Zealand team for the 2012 IMO will be chosen from this squad.

There are two sets of problems: junior division and senior division.

- If you are currently in year 12, **or** you have been a member of the NZIMO training squad, then you may **only** attempt the senior problems.
- If you are currently in year 11 or below, **and** you have never been a member of the NZIMO training squad, then you may attempt **both** sets of problems (and your results from both sets will be taken into account in the selection process).

Note: Students in year 11 or below who have previously attended a January camp, but not been a member of the NZIMO training squad, are no longer required to do just the senior problems.

General instructions:

- Although some problems seem to require only a numerical answer, in order to receive full credit for the problem a complete justification must be provided. In fact, an answer alone will only be worth 1 point out of 7.
- You may not use a calculator, computer or the internet to assist you in solving the problems.
- All solutions must be entirely your own work.
- We do not expect many, if any, perfect submissions. So, please submit all the solutions and partial solutions that you can find.

Students submitting solutions should be intending to remain in school in 2012 and should also hold New Zealand Passports or have New Zealand Resident status. To be eligible for the 2012 IMO you must have been born on or after 20 July 1992, and must not be formally enrolled in a University or similar institution prior to the IMO.

Your solutions, together with a completed Registration Form (overleaf), should be sent to

Michael Albert, Department of Computer Science, University of Otago, PO Box 56,
Dunedin 9054

arriving no later than 10 August 2011. You will be notified whether or not you have been selected for the Camp by 30 September 2011.

If you have any questions, please contact Michael Albert (michael.albert@cs.otago.ac.nz, (03) 479 8586) or Chris Tuffley (c.tuffley@massey.ac.nz, (06) 356 9099 x3573).

June 2011
www.mathsolympiad.org.nz

Registration Form
NZMOC Camp Selection Problems 2011

Name: ______________________________

Gender: male/female

School year level in 2011: ______________________________

Home address: ______________________________

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Email address: ______________________________

Home phone number: ______________________________

School: ______________________________

School address: ______________________________

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Principal: ______________________________

HOD Mathematics: ______________________________

Do you intend to take part in the camp selection problems for any other Olympiad camp? yes/no

If so, and if selected, which camp would you prefer to attend? ______________________________

Have you put your name forward for a Science camp or any other camp in January? yes/no

Are there any criminal charges, or pending criminal charges against you? yes/no

Some conditions are attached to camp selection. You must be:

- Born on or after 20 July 1992
- Studying in 2012 at a recognised secondary school in NZ
- Available in July 2012 to represent NZ overseas as part of the NZIMO team if selected.
- A NZ citizen or hold NZ resident status.

Declaration: I satisfy these requirements, have worked on the questions without assistance from anyone else, and have read, understood and followed the instructions for the January camp selection problems. I agree to being contacted through the email address I have supplied.

Signature: ______________________________ Date: ______________________________

Attach this registration form to your solutions, and send them to

> Michael Albert, Department of Computer Science, University of Otago, PO Box 56, Dunedin 9054,

arriving no later than 10 August 2011.

New Zealand Mathematical Olympiad Committee

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Due: 10 August 2011

Junior division

J1. A three by three square is filled with positive integers. Each row contains three different integers, the sums of each row are all the same, and the products of each row are all different. What is the smallest possible value for the sum of each row?

J2. Let an acute angled triangle ABC be given. Prove that the circles whose diameters are AB and AC have a point of intersection on BC.

J3. There are 16 competitors in a tournament, all of whom have different playing strengths and in any match between two players the stronger player always wins. Show that it is possible to find the strongest and second strongest players in 18 matches.

J4. Find all pairs of positive integers m and n such that:

$$(m+1)! + (n+1)! = m^2n^2.$$

(Note: $k! = 1 \times 2 \times 3 \times \cdots \times k$.)

J5. Let a square $ABCD$ with sides of length 1 be given. A point X on BC is at distance d from C, and a point Y on CD is at distance d from C. The extensions of: AB and DX meet at P, AD and BY meet at Q, AX and DC meet at R, and AY and BC meet at S. If points P, Q, R and S are collinear, determine d.

J6. Find all pairs of non-negative integers m and n that satisfy

$$3 \times 2^m + 1 = n^2.$$

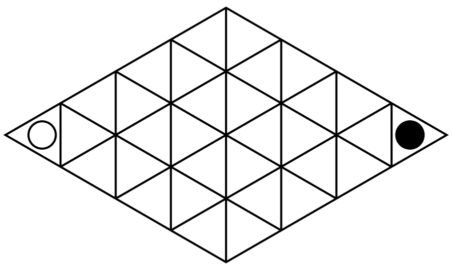
Senior division

S1. Find all pairs of positive integers m and n such that

$$m! + n! = m^n.$$

S2. In triangle ABC, the altitude from B is tangent to the circumcircle of ABC. Prove that the largest angle of the triangle is between 90° and 135°. If the altitudes from *both* B and from C are tangent to the circumcircle, then what are the angles of the triangle?

S3. Chris and Michael play a game on a board which is a rhombus of side length n (a positive integer) consisting of two equilateral triangles, each of which has been divided into equilateral triangles of side length 1. Each has a single token, initially on the leftmost and rightmost squares of the board, called the "home" squares (the illustration shows the case $n = 4$).



A move consists of moving your token to an adjacent triangle (two triangles are adjacent only if they share a side). To win the game, you must either capture your opponent's token (by moving to the triangle it occupies), or move on to your opponent's home square. Supposing that Chris moves first, which, if any, player has a winning strategy?

S4. Let a point P inside a parallelogram $ABCD$ be given such that $\angle APB + \angle CPD = 180^\circ$. Prove that

$$AB \cdot AD = BP \cdot DP + AP \cdot CP$$

(here AB, AD etc. refer to the lengths of the corresponding segments).

S5. Prove that for any three distinct positive real numbers a, b and c:

$$\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} > 8abc.$$

S6. Consider the set G of 2011^2 points (x, y) in the plane where x and y are both integers between 1 and 2011 inclusive. Let A be any subset of G containing at least $4 \times 2011 \times \sqrt{2011}$ points. Show that there are at least 2011^2 parallelograms whose vertices lie in A and all of whose diagonals meet at a single point.