# New Zealand Mathematical Olympiad Committee

## 2011 Squad Training Test

*Problems*

1. Given a positive integer $n$, find the minimal integer $k$ such that for any $n$ real numbers $c_i$ and points $A_i = (x_i, y_i)$ on the plane with no three collinear, there exists a polynomial $P(x, y)$ such that $P(x_i, y_i) = c_i$ for $i = 1, \ldots, n$, and the degree of $P$ is not greater than $k$.

   (*Note:* the degree of a multivariate polynomial is defined as the maximum sum of the exponents in any single term in the polynomial, for example, the degree of $P(x, y) = x^3y^4 + x^6$ is 7)

2. In a boarding school, 512 students learn a total of 9 subjects. Out of the 9 subjects, every student is interested in some subjects and not interested in the others. Furthermore, for any two students, their sets of interests are distinct (in particular, exactly one student will not be interested in any subjects). These students live in 256 double rooms; two students sharing a room are called *roommates* (this isn't an entirely new concept). Prove that all the students can be arranged in a circle such that

   (a) every two roommates are adjacent in the circle and

   (b) for every two adjacent students who are not roommates, one of them is interested in all the subjects which the other one is interested in, and is furthermore interested in exactly one more subject.

3. A quadrilateral $ABCD$ is inscribed into a circle. Its diagonals intersect at point $K$. Let $M_1, M_2, M_3, M_4$ be the midpoints of arcs $AB, BC, CD, DA$ (not containing the other vertices) respectively. Let $I_1, I_2, I_3, I_4$ be the incentres of triangles $ABK, BCK, CDK, DAK$ respectively. Prove that the lines $M_1I_1, M_2I_2, M_3I_3, M_4I_4$ are concurrent.

4. Given $n \geq 3$ pairwise relatively prime positive integers, it is known that while dividing the product of any $n - 1$ of them by the remaining integer, the remainder equals a constant integer $r$. Prove that $r \leq n - 2$.

5. A polynomial $P(x)$ of degree $n \geq 3$ has $n$ real roots $x_1 < x_2 < \ldots < x_n$ such that $x_2 - x_1 < x_3 - x_2 < \ldots < x_n - x_{n-1}$. Prove that the maximum of the function $y = |P(x)|$ on the interval $[x_1, x_n]$ is attained on the interval $[x_{n-1}, x_n]$.

*Sunday 24 April 2011*

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