



2011 Squad Assignment Two

*Geometry*

**Due: Monday 28th February 2011**

1. Find all possible values of the quotient

$$\frac{r + \rho}{a + b}$$

where  $r$  and  $\rho$  are respectively the radii of the circumcircle and incircle of the right triangle with legs  $a$  and  $b$ .

2. Let  $ABC$  be an isosceles triangle with  $|AB| = |AC|$  and  $P, Q$  are interior points of  $AB$  and  $AC$  respectively. Prove that the circumcircle of  $\triangle APQ$  passes through the circumcentre of  $\triangle ABC$  if and only if  $|AP| = |CQ|$
3. Let  $ABCD$  be a rhombus and let a tangent of its incircle cut the interior of the sides  $BC$  and  $CD$ , and denote  $R, S$  the intersections of the tangent with the lines  $AB, AD$  respectively. Prove that the value of  $|BR| \cdot |DS|$  is independent of the choice of the tangent.
4. In an acute-angled triangle  $ABC$ ,  $M$  is the midpoint of side  $BC$ , and  $D, E$  and  $F$  the feet of the altitudes from  $A, B$  and  $C$ , respectively. Let  $H$  be the orthocentre of triangle  $ABC$ ,  $S$  the midpoint of  $AH$ , and  $G$  the intersection of  $FE$  and  $HA$ . If  $N$  is the intersection of the line segment  $AM$  and the circumcircle of triangle  $BCH$ , prove that  $\angle HMA = \angle GNS$ .
5. Points  $C, D, E$  and  $F$  lie on a circle with centre  $O$ . The two chords  $CD$  and  $EF$  intersect at a point  $N$ . The tangents at  $C$  and  $D$  intersect at  $A$ , and the tangents at  $E$  and  $F$  intersect at  $B$ . Prove that  $ON \perp AB$ .
6. Can the four incentres of the four faces of a tetrahedron be coplanar?
7. Let  $O$  be the circumcentre of an acute-angled triangle  $ABC$ . A line through  $O$  intersects the sides  $CA$  and  $CB$  at points  $D$  and  $E$  respectively, and meets the circumcircle of triangle  $ABO$  again at point  $P \neq O$  inside the triangle. A point  $Q$  on side  $AB$  is such that

$$\frac{AQ}{QB} = \frac{DP}{PE}.$$

Prove that  $\angle APQ = 2\angle CAP$ .

*February 10, 2011*

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