New Zealand Mathematical Olympiad Committee

2011 April Problems

These problems are intended to help students prepare for the 2011 camp selection problems, used to choose students to attend our week-long residential training camp in January.

The solutions will be posted in about one month's time, but can be obtained before then by email if you write to one of us with evidence that you've tried the problems seriously.

Good luck!

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1. Points A,B,C lie on a circle. Line PB is tangent to the circle at B. Let PA_1 and PC_1 be the perpendiculars from P to lines AB and BC respectively (A_1 and C_1 lie on lines AB and BC respectively). Prove that A_1C_1 is perpendicular to AC.
2. Does there exist a positive integer k such that all the positive integers from 1 to k can be partitioned into two sets, and all the numbers in each set can be written down one after another (in some order without spaces) to form two new numbers, so that these two numbers are equal.
3. Nine skiers participated in a race. They started one by one, and each skier completed the race with a constant speed (which could be different for different skiers). Determine if it could happen that each skier participated in an overtaking exactly four times. (In each overtaking, exactly two skiers participated: the one who overtakes and the one who is overtaken.)
4. Let 100 pairwise distinct real numbers be arranged in a circle. Prove that there exist four consecutive numbers along the circle such that the sum of the middle two numbers is strictly less than the sum of the other two numbers.

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