New Zealand
Maths Olympiad Committee
Camp 2009
Problem Set 2

1. A right angled triangle has hypotenuse of length 6 m. A square is constructed, one of whose sides forms part of the hypotenuse, and whose other two vertices are on the legs of the triangle. Show that the area of this square is at most 4 m^2, and determine for which triangles equality can occur.

2. Let a, b and c be real numbers. Prove that at least one of $(a+b+c)^2-9ab$, $(a+b+c)^2-9bc$, $(a+b+c)^2-9ca$ is non-negative.

3. Show that among any eight distinct composite integers less than or equal to 360, there are at least two which are not relatively prime.

4. The sides and longest diagonal of a three-dimensional rectangular box all have integer lengths. If two of the sides have lengths 9 and 12, what are the possible lengths of the third side?

5. An acute angled triangle ABC is given. Determine the set of all points P inside the triangle such that:
$$\frac{\angle APB}{\angle ACB}, \frac{\angle BPC}{\angle BAC}, \frac{\angle CPA}{\angle CBA} \leq 2.$$

6. Let $A = a_1, a_2, \ldots, a_{2009}$ be a sequence of positive integers. Let m_A be the number of three element subsequences a_i, a_j, a_k with $1 \leq i < j < k \leq 2009$ and $a_j = a_i + 1$ and $a_k = a_j + 1$. Among all such sequences A, find the maximum possible value of m_A.