

New Zealand Maths Olympiad Committee
2009 Maths Gymnastics
Christchurch, Thursday 14 January

1. Let n be a positive integer. Prove that $4^n + 15n - 1$ is divisible by 9.
2. Suppose α , β and γ are the three angles of a given triangle and

$$\sin(\alpha) = 2 \sin(\beta) \cos(\gamma)$$

Prove that the triangle is isosceles.

3. What are the last two digits of the number

$$7^{999}$$

4. Suppose we have a convex polygon with 1000 sides. There are 500 points inside of the polygon (with no 3 of these points being collinear). We then cut this polygon to obtain triangles using these 1500 points as vertices. How many triangles will we have?
5. Find the value of

$$\cot 70^\circ + 4 \cos 70^\circ$$