New Zealand Mathematical Olympiad Committee

Sample Algebra Problems

by Ross Atkins

1. Let $a_1, a_2, a_3, \ldots$ be an infinite sequence such that

$$a_{n+1} = a_n - a_{n-1}.$$

   Given $a_1 = 2$, determine all possible values of a_{2017}.

2. For any x, y and z, show that

$$x^2 + y^2 + z^2 \geq xy + yz + zx.$$

3. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(3x + f(0)) = 3x^2$$

   for all real x.

4. Suppose $p(x)$ is a polynomial of degree n, such that for $k = 0, 1, 2, 3, \ldots, n$ we have

$$p(k) = \frac{k}{k+1}.$$

   Determine the value of $p(n+1)$. *(express your answer in terms of n)*