



New Zealand Mathematical Olympiad Committee

Auckland Maths Workshop

November 20th, 6:00pm to 8:00pm

University of Auckland, rooms 303-G14 and 303-G15

Problems

1. Solve the equation $\log_x(x+2) = 2$.
2. Grace has one more coin than William. Both players throw all of their coins simultaneously and observe the number that come up heads. Assuming all the coins are fair, what is the probability that Grace obtains strictly more heads than William?
3. Let $ABCD$ be any convex quadrilateral. Let M , N , P and Q be the midpoints of AB , BC , CD and DA respectively. Prove that $MNPQ$ is a parallelogram.
4. What is the remainder when 2017^{2018} is divided by 2018?
5. Suppose a , b are integers such that both $2a + 3b$ and $3a - 2b$ are the squares of positive integers. What is the smallest possible value of these squares?
6. Show that any set of 10 integers contains a subset whose sum is a multiple of 10.
7. Let M be the midpoint of side BC of triangle ABC . Let D be the foot of the altitude from A . If $\angle DAB = \angle CAM$, show that triangle ABC is either isosceles or right-angled.
8. Let a, b, c, d be any four distinct integers such that $a > b > c > d > 1$. Show that if $ad = bc$, then $a^2 + b^2 + c^2 + d^2$ is composite.