New Zealand Mathematical Olympiad Committee

Maths Workshop (Christchurch)

Monday September 2nd, 6:00pm to 8:00pm

University of Canterbury, Erskine rooms 443 and 446

Problems

1. Each day Jeffrey earns \$3 for washing the dishes. He can earn \$5 instead by also sweeping the kitchen. After ten days, Jeffrey has earned a total of \$36. On how many of these days did Jeffrey sweep the kitchen?

2. Josie and Ross play a game in which two dice are rolled every turn. The value of a turn is the sum of the two dice rolled on that turn. Josie wins as soon as a turn has value 12. Ross wins if two consecutive turns both have value 7. What is the probability that Josie wins before Ross?

3. Let $ABCD$ be a parallelogram, and let M be the midpoint of CD. Let P be the intersection of AM and BD. What is the ratio $BP : PD$?

4. Does there exist a perfect square such that the last 3 decimal digits (the units, tens and hundreds digits) are the same and non-zero? What about a perfect square such that the last 4 decimal digits are the same and non-zero?

5. A number is called *nice* if each digit (other than the left-most digit) is either one more or one less than some digit to the left of it. How many nice ten-digit numbers are there with each of the digits (from 0 to 9) appearing exactly once?

6. Let a, b, c be positive real numbers such that $abc = 1$. Show that

$$a + b + c \leq a^4 + b^4 + c^4.$$