



New Zealand Mathematical Olympiad Committee

Maths Workshop (Auckland Central)

Friday August 2nd, 6:15pm to 8:15pm

University of Auckland, rooms 303-G15 and 303-G16

Problems

1. What is the value of

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{23 \times 25}$$

2. How many ways can 8 be written as an ordered sum of 4 positive integers?
(for example $1+2+4+1$ is considered different to $4+1+1+2$)
3. Let ABC be any triangle. Prove that the angle bisectors of $\angle A$ and $\angle B$ and $\angle C$ are concurrent (that they meet at a single point).
4. Does there exist an integer k such that $\log_{10}(1 + 39k)$ is also an integer?
5. Find all pairs of real numbers x and y such that $x^2 + y^2 = 1$ and

$$\frac{1}{x} + \frac{1}{y} = 2\sqrt{2}.$$

6. A sequence is called *good* if no four terms (not necessarily consecutive) are in increasing order, and no four terms are in decreasing order. What is the length of the longest good sequence you can find?
7. A wooden triangle has side lengths equal to 1, 1 and $\sqrt{2}$. The triangle is to be sawn into two pieces of equal area. Find the length of the shortest straight cut which achieves this.
8. Find all triplets (a, b, c) of positive integers such that

$$a \mid bc + 1 \quad \text{and} \quad b \mid ca + 1 \quad \text{and} \quad c \mid ab + 1.$$