Maths Workshop

August 13th, 6:00pm to 8:00pm.
Auckland University, 303-B09.

Problems

1. If a, b, c and d are positive real numbers, prove that
$$\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd}.$$

2. At a party of 10 people, everyone shakes hands with everyone else. How many handshakes take place.

3. Let ABC be any triangle. Prove that the angle bisectors of $\angle ABC$, $\angle BCA$ and $\angle CAB$ are concurrent at a point.

4. Find the smallest positive integer which leaves a remainder of 2 when divided by 48, and leaves a remainder of 9 when divided by 49.

5. Find all real numbers x such that
$$\sqrt{x+2\sqrt{x-1}} - \sqrt{x-2\sqrt{x-1}} = 2.$$

6. Ross owns a 7m by 14m property containing four trees (each with diameter 1m). Prove that if a playhouse is a 2m by 4m rectangle, then Ross can fit four playhouses on his property without removing any trees.

7. Let ABC be any triangle and let M is the midpoint of BC. Prove that
$$2AM < AB + AC.$$

8. You have 100 doors in a row that are all initially closed. You make 100 passes by the doors starting with the first door every time. The first time through you visit every door and toggle the door (if the door is closed, you open it, if its open, you close it). The second time you only visit every 2^{nd} door (doors $2, 4, 6, \ldots$). The third time, every 3^{rd} door (doors $3, 6, 9, \ldots$), and so on, until (on the 100^{th} pass) you only visit the 100^{rd} door. How many doors are open in the end?