



New Zealand Mathematical Olympiad Committee

Maths Workshop (Auckland Central)

February 12th, 6:15pm to 8:15pm

University of Auckland, rooms 303-G14 and 303-G15

Problems

1. A domain contains 9 red balls and 7 yellow balls. Whenever two balls of the same colour collide, they merge into a single yellow ball. Whenever two balls of different colour collide, they merge into a single red ball. What will be the colour of the last ball?
(available online: <http://www.gustygames.co.nz/challenges/Smileys.html>)
2. Show that $n^4 + 4$ is composite for all integers $n > 1$.
3. An equilateral triangle is inscribed in a circle of area 1. Then the second circle is inscribed in the triangle. Find the area of the second circle.
4. I am thinking of a number with units-digit equal to 6. If this units digit 6 is removed, and placed at the beginning of my number, then the number becomes 4 times larger. What is the smallest number I could be thinking of?
5. Quadrilateral $ABCD$ has sidelengths $AB = 1$, $BC = 5$, $CD = 5$ and $DA = 7$. What is the maximal possible area of quadrilateral $ABCD$?
6. Show that there are infinitely many integer solutions (x, y) to the equation

$$x^2 - 3y^2 = 1.$$

7. A plane is divided by 3027 straight lines such that neither two of them are parallel and neither three of them intersect at one point. Prove that among the pieces of the plane obtained as a result of such division there are at least 2018 triangular pieces.
8. Find the 100th digit to the right of the decimal point in the decimal representation of

$$(1 + \sqrt{2})^{2019}.$$