Auckland Maths Workshop

December 11th, 6:00pm to 8:00pm

University of Auckland, rooms 303-G15 and 303-G16

Problems

1. Given $\sin A + \sin^2 A = 1$, what is the value of $\cos^2 A + \cos^4 A$?

2. The integers from 1 to 2018 are written on the blackboard. Two randomly chosen numbers are erased and replaced by their difference giving a sequence with one less number. This process is repeated until there is only one number remaining. Is the remaining number even or odd?

3. Let ABC be any triangle and let D, E and F be the midpoints of AB, BC and CA. Let X be the point on BC such that AX is perpendicular to BC. Prove that X lies on the circumcircle of DEF.

4. Show that, if n is an integer, $n^2 + 11n + 2$ is not divisible by 12769.

5. The sum of three numbers is 6, and their product is 2. If the sum of their squares is 18, then what is the sum of their cubes?

6. The NZMO Committee consists of 12 members. Each pair of members of the committee either mutually like each other, or mutually dislike each other. Each member likes exactly 6 other members. Is it necessarily possible to sit all 12 members around a round table so that any two adjacent members like each other?

7. Let H be the orthocentre of triangle ABC (H is the point inside triangle ABC such that $AH \perp BC$, and $BH \perp AC$ and $CH \perp AB$). Prove that the midpoints of AB, AC, AH, BC, BH and CH form a cyclic hexagon.

8. Find the smallest positive integer that has exactly 50 positive divisors. (The term divisor includes 1 and the number itself. So, for example, 9 has three positive divisors.)