



New Zealand Mathematical Olympiad Committee

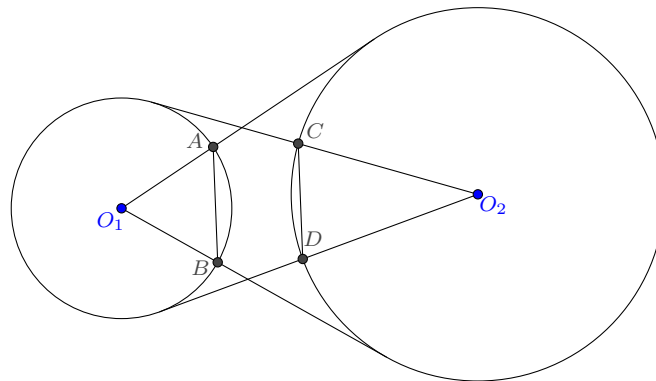
## Maths Workshop (Auckland East)

Friday May 10th, 6:00pm to 8:00pm

*Macleans College, Kupe House*

### Problems

1. Solve the equation  $3^x + 9^x = 27^x$ .
2. How many rectangles can you form using only the lines on a regular  $8 \times 8$  chessboard?
3. Let  $\Gamma_1$  and  $\Gamma_2$  be any two circles with centres  $O_1$  and  $O_2$  respectively. Let  $A$  and  $B$  be the intersection of  $\Gamma_1$  with the tangents from  $O_1$  to  $\Gamma_2$ . Similarly let  $C$  and  $D$  be the intersection of  $\Gamma_2$  with the tangents from  $O_2$  to  $\Gamma_1$ . Show that chords  $AB$  and  $CD$  have equal length.



4. What comes next: 2, 3, 4, 7, 8, 15, 24, 60, ?
5. Find the value of the product
$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \cdots \times \frac{k^3 - 1}{k^3 + 1} \times \cdots \times \frac{999999}{1000001}$$
6. In how many ways, counting ties, can eight horses cross the finishing line?  
(eg. two horses, A and B, can finish in three ways: A wins, B wins, A and B tie.)
7. The towns of Alpha, Beta, and Gamma are equidistant from each other. If a car is three miles from Alpha and four miles from Beta, what is the maximum possible distance of the car from Gamma?
8. Is it true that  $(n! + 1)$  is composite for infinitely many positive integers  $n$ ?