



New Zealand Mathematical Olympiad Committee

Auckland Maths Workshop

October 9th, 6:15pm to 8:15pm

University of Auckland, rooms 303-G14 and 303-G15

Problems

1. Let a, b, c, d and e be any 5 odd integers. Prove that the equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

has no integer solutions.

2. How many ways are there to rearrange the letters of the word “MISSISSIPPI”?
3. Let $ABCD$ be a quadrilateral such that there is a circle which is tangent to all four sides. Prove that $AB + CD = AD + BC$.
4. Find all prime numbers p such that $5p + 2$ is a square number.
5. There are n identical cars on a circular track. Among all of them, they have exactly enough fuel for one lap. Show that there is a car which can complete a lap by collecting fuel from the other cars on its way around.
6. A fair coin is tossed 10 times. What is the probability that no two consecutive heads appear?
7. A convex quadrilateral $ABCD$ has sides $AB = 2$, $BC = 8$, $CD = 6$, and $DA = 7$. The incircles of triangles ABC and ACD are tangent to AC at points E and F . Find EF .
8. Peter thinks of two single digit positive integers a and b . He tells the sum $a + b$ to Grace, and the product ab to William. Grace and William then have the following conversation:
 - William: I do not know what the two numbers are.
 - Grace: I also do not know what the two numbers are.
 - William: Now I do know what a and b are.
 - Grace: So do I.

What were a and b ?