New Zealand Mathematical Olympiad Committee

Maths Workshop (Auckland East)

March 8th, 6:00pm to 8:00pm

Macleans College, Kupe House

Problems

1. In a race with 30 runners where 8 trophies will be given to the top 8 runners (the trophies are distinct: first place, second place, etc), how many ways can this be done?
2. Solve the equation $x^2 - x - \cos y + \frac{5}{4} = 0$.
3. ABC is right-angled at A. The angle bisector from A meets BC at D. If $CD = 1$ and $BD = AD + 1$, find the lengths of AC and AD.
4. Find all ordered pairs (a, b) of positive integers such that $|3^a - 2^b| = 1$.
5. What is the sum of positive divisors of 2310.
6. Triangle ABC is right-angled at C. Points X and Y are on sides BC and AC. Lines AX and BY are the angle bisectors of $\angle CAB$ and $\angle ABC$. If $AX = 9$ and $BY = 8\sqrt{2}$, then find AB.
7. The first 100 positive integers are arbitrarily divided into two groups of 50 numbers each. The numbers in the first group are sorted in ascending order: $a_1 < a_2 < \cdots < a_{50}$; the numbers in the second group are sorted in descending order: $b_1 > b_2 > \cdots > b_{50}$. What is the maximum and minimum possible values of the sum $|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_{50} - b_{50}|$?
8. Suppose $P(x)$ is a polynomial of degree n such that $P(k) = \frac{k}{k+1}$ for $k = 0, 1, 2, \ldots, n$. What is the value of $P(n+1)$?