



New Zealand Mathematical Olympiad Committee

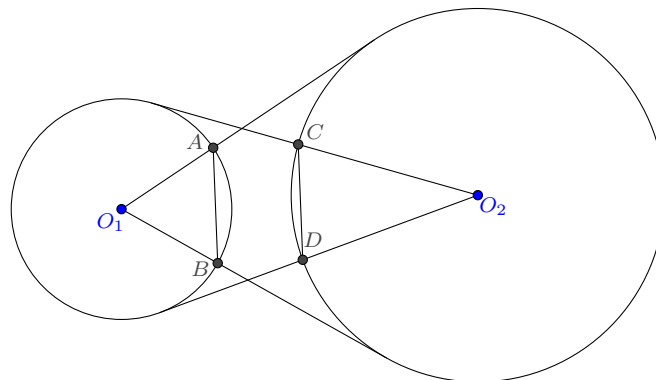
Maths Workshop (Auckland Central)

Monday May 6th, 6:15pm to 8:15pm

University of Auckland, rooms 303-G14 and 303-G15

Problems

1. Solve the equation $3^x + 9^x = 27^x$.
2. How many rectangles can you form using only the lines on a regular 8×8 chessboard?
3. Let Γ_1 and Γ_2 be any two circles with centres O_1 and O_2 respectively. Let A and B be the intersection of Γ_1 with the tangents from O_1 to Γ_2 . Similarly let C and D be the intersection of Γ_2 with the tangents from O_2 to Γ_1 . Show that chords AB and CD have equal length.



4. What comes next: 2, 3, 4, 7, 8, 15, 24, 60, ?
5. Find the value of the product
$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \cdots \times \frac{k^3 - 1}{k^3 + 1} \times \cdots \times \frac{999999}{1000001}$$
6. In how many ways, counting ties, can eight horses cross the finishing line?
(eg. two horses, A and B, can finish in three ways: A wins, B wins, A and B tie.)
7. The towns of Alpha, Beta, and Gamma are equidistant from each other. If a car is three miles from Alpha and four miles from Beta, what is the maximum possible distance of the car from Gamma?
8. Is it true that $(n! + 1)$ is composite for infinitely many positive integers n ?