New Zealand Mathematical Olympiad Committee

Maths Workshop (Auckland Central)

Tuesday April 2nd, 6:15pm to 8:15pm

University of Auckland, rooms 303-G13 and 303-G15

Problems

1. After returning from a holiday to the USA, Megan has some American coins: A 25c coins (quarters) and B 10c coins with a total value of \$1.95, where A and B are both counting numbers. How many different values of A can Megan have?
2. Let x be a real number. Which is greater, $\sin(\cos x)$ or $\cos(\sin x)$?
3. A pair of circles intersect at points A and B. A line is tangent to both circles, at points C and D. Prove that the intersection of AB and CD is the midpoint of CD.
4. Let x, y and n be positive integers such that $x(x+1) = y^n$. Is it possible that $n > 1$.
5. Let $a_1, a_2, a_3, \ldots$ be a strictly increasing sequence of positive integers such that:
   - $a_2 = 2$.
   - $a_{mn} = a_m a_n$ for m, n relatively prime (multiplicative property).

   Prove that $a_n = n$, for all $n \geq 1$.
6. Let A and B be the intersection points of circles Γ_1 and Γ_2. Suppose $S \in \Gamma_1$ and $T \in \Gamma_2$ such that ST is a common tangent of Γ_1 and Γ_2, and A lies inside triangle BST. Line AT intersects Γ_1 at two points A and C. Prove that BS is the angle bisector of $\angle CBT$.
7. Show that any two elements (both greater than one) drawn from the same row of Pascal's triangle have greatest common divisor greater than one.
8. Find the value of $f(1) + f(2) + f(3) + \cdots + f(60)$, where

$$f(x) = \frac{4x + \sqrt{4x^2 - 1}}{\sqrt{2x+1} + \sqrt{2x-1}}.$$