



Maths Workshop (Christchurch)

Monday April 1st, 6:00pm to 8:00pm

University of Canterbury, Erskine room 446

Problems

1. After returning from a holiday to the USA, Megan has some American coins: A 25c coins (quarters) and B 10c coins with a total value of \$1.95, where A and B are both counting numbers. How many different values of A can Megan have?
2. Let x be a real number. Which is greater, $\sin(\cos x)$ or $\cos(\sin x)$?
3. A pair of circles intersect at points A and B . A line is tangent to both circles, at points C and D . Prove that the intersection of AB and CD is the midpoint of CD .
4. Let x , y and n be positive integers such that $x(x+1) = y^n$. Is it possible that $n > 1$.
5. Let a_1, a_2, a_3, \dots be a strictly increasing sequence of positive integers such that:
 - $a_2 = 2$.
 - $a_{mn} = a_m a_n$ for m, n relatively prime (multiplicative property).

Prove that $a_n = n$, for all $n \geq 1$.

6. Let A and B be the intersection points of circles Γ_1 and Γ_2 . Suppose $S \in \Gamma_1$ and $T \in \Gamma_2$ such that ST is a common tangent of Γ_1 and Γ_2 , and A lies inside triangle BST . Line AT intersects Γ_1 at two points A and C . Prove that BS is the angle bisector of $\angle CBT$.
7. Show that any two elements (both greater than one) drawn from the same row of Pascal's triangle have greatest common divisor greater than one.
8. Find the value of $f(1) + f(2) + f(3) + \dots + f(60)$, where

$$f(x) = \frac{4x + \sqrt{4x^2 - 1}}{\sqrt{2x+1} + \sqrt{2x-1}}.$$